8. Loop of wire (7 pts)

1) At the distance r from the current I_0, the magnetic induction $B = \dfrac{\mu_0 I_0}{2\pi r}$. Then, the flux through the contour $\Phi = \int_l^{l+a} Bb\,dx = \int_l^{l+a} \dfrac{b\mu_0 I_0}{2\pi x}\,dx = \dfrac{b\mu_0 I_0}{2\pi} \ln\dfrac{a+l}{l}$.

Alternatively, we can find it using the graph by determining the area S under the curve, from $r = r_1 = 0{,}01\,\text{m}$ to $r = r_2 = 0{,}04\,\text{m}$: $S \approx 0{,}28\,\text{mT}\cdot\text{m}$, further, $\Phi = Sb = 280\,\mu\text{Wb}$.

2) After switching off the current, the flux through the tends to zero. From the Ohm's law $R\dfrac{dq}{dt} = \dfrac{d\Phi}{dt}$, hence $Rdq = d\Phi$, i.e. $RQ = \Delta\Phi = \Phi$. Finally, $Q = \Phi/R = 280\,\mu\text{C}$.

3) We calculate the force as difference between the forces at the two loop segments parallel to the straight line: $F_1 = biB_1$ and $F_2 = biB_2$, where $i = R^{-1}\dfrac{d\Phi}{dt}$. So, $dp = (F_1 - F_2)\,dt = bR^{-1}(B_1 - B_2)\,d\Phi$. Using $B_1 = \dfrac{\mu_0 I}{2\pi l}$ and $B_1 = \dfrac{\mu_0 I}{2\pi(l+a)}$, we end up with $dp = \dfrac{b}{R}\,\dfrac{\mu_0 I}{2\pi}\,\dfrac{a}{l(l+a)}\,d\Phi$. Using the result of first question, $d\Phi = \dfrac{b\mu_0}{2\pi}\ln\dfrac{a+l}{l}\,dI$, i.e. $dp = \left(\dfrac{b\mu_0}{2\pi}\right)^2 \dfrac{a}{Rl(l+a)} \ln\dfrac{a+l}{l}\,I\,dI$. Finally, $p = \dfrac{a(b\mu_0 I_0)^2}{8\pi^2 Rl(l+a)} \ln\dfrac{a+l}{l} \approx 2.08\cdot 10^{-6}\,\text{kg}\cdot\text{m/s}^2$.

The same result could have been obtained using the graph and approach used in the alternative solution of the question 1.