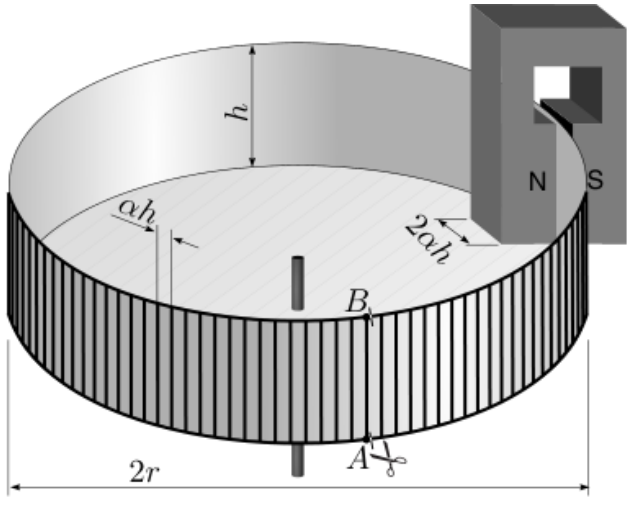
2. Magnetic break (12 points)

Foucault currents can be used to decelerate moving metal objects, e.g. a rotating disc. Consider the following simple model. For the sake of simplicity, the disc is substituted by a "circular railway", see Figure. Plastic "can" of radius $r = 15\,\mathrm{cm}$, mass $m = 100\,\mathrm{g}$, and height $h = 1\,\mathrm{cm}$ consists of a homogeneous disc (the bottom of the "can"), and of a much thinner cylindrical wall. Along the edges of the cylinder, there are two wire rings ("rails"), which are connected with a set of parallel wire bars ("sleepers"). Both the "sleepers" and "rails" are made of a copper wire of diameter $\delta = 0{,}2\,\mathrm{mm}$; the distance between the "sleepers" $L = \alpha h$, where $\alpha = 0{,}3$. Frictionless rotation of the system is decelerated with an homogeneous magnetic field ($B = 1\,\mathrm{T}$) in the slit between the poles of a permanent magnet, see Figure. Assume that the homogeneous field fills a region of rectangular cross-section, equal to the area between three subsequent "sleepers" (i.e. of size $h \times 2\alpha h$). Outside of that region, the field is negligible. The specific resistance of the copper $\rho = 1.724 \cdot 10^{-8}\,\Omega\mathrm{m}$.



Suppose the system is motionless.

1) Find the resistance R of a single "sleeper" (1 pt).

2) Suppose the "rails" are cut near the end-points of a certain "sleeper" A and B. Prove that the resistance between A and B is approximately given by $R_0 = R[\sqrt{\alpha(\alpha+2)} - \alpha]$ (2 pts).

Now suppose the system is rotating with an angular speed $\omega = 1\,\mathrm{rad/s}$, and the "rails" are uncut.

3) Sketch an equivalent DC circuit, so that the currents through the resistors are equal to the currents in the respective "railway" elements: "sleepers" and "rail" segments (between subsequent "sleepers"; 2 pts).

4) Using the above obtained results, prove that the (Joule) dissipation power is given by formula $P = kB^2\omega^2/R$, and express the constant k (3 pts).

5) Find the decelerating torque M (2 pts).

6) Prove that the angular speed will vanish as $\omega = \omega_0 e^{-t/\tau}$, and determine the time constant τ (2 pts).