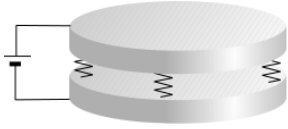
6. Mechano-electrical oscillator (7 points)

Mechanical and electrical processes are sometimes strongly coupled. Very important examples are systems containing piezoelectric materials, e.g. quartz resonator. Here we investigate a somewhat simpler situation.

There are two metal plates with area S and mass m. One plate is situated atop of the other one. Plates are connected to each other with springs, whose total spring constant is k and what are made of insulator. The lower plate is mounted on a steady base. Equilibrium distance between the plates is X_0.



1) Let us assume that there is a small vertical shift x of the upper plate from its equilibrium position. Derive acceleration $\ddot{x}$ of x in terms of system parameters. What is the angular frequency ω_0 of the small vertical oscillations of the upper plate (1 pts)?

2) Plates are now connected to a constant high voltage source, so that they form a capacitor. Electrostatic force between the plates causes an additional shift of the upper plate. The equilibrium distance between the plates is now X_1. Derive expressions of the electrical attractive force F_e and voltage applied to the plates U in terms of X_0, X_1, S, m and k (2 pts).

3) System is set to oscillate again, keeping voltage U constant. Let x stand still for the shift from the equilibrium position. Derive an expression for the acceleration $\ddot{x}$ of x in terms of X_0, X_1, S, m, k and shift x. What is the angular frequency ω_1 of upper plate's small vertical oscillations (2 pts)?

4) Let us modify the situation of the previous question and connect an inductor with inductance L in series to the capacitor and voltage source. We describe the situation in terms of plate shift x and capacitor's charge q. Derive expressions for the accelerations $\ddot{x}$ and $\ddot{q}$ in terms of X_0, X_1, S, m, k, x and q. Which angular frequencies of harmonic oscillation are possible in the system? (2 pts)