4. Water pump

1) Let us consider the process in the system, rotating together with the tank. Then, there is a potential energy related to the centrifugal force:

$$U_c = \int_0^r \omega^2 r \, \mathrm{d}r = \frac{1}{2}\omega^2 r^2.$$

So, the pressure

$$p_2 = p_0 - \rho g h + \frac{1}{2}\omega^2 r^2.$$

2) From the Bernoulli formula,

$$\frac{1}{2}\rho u^2 = p_2 - p_0 = \frac{1}{2}\omega^2 r^2 - \rho g h,$$

hence the squared velocity in the rotating reference system

$$u^2 = \omega^2 r^2 - 2gh.$$

The laboratory speed

$$v_2 = u^2 + \omega^2 r^2 = 2\left(\omega^2 r^2 - gh\right),$$

i.e.

$$v_2 = \sqrt{2\left(\omega^2 r^2 - gh\right)}.$$

3) The point of lowest pressure p_m inside the pump is the upmost point of the tube. Using the Bernoulli formula,

$$p_0 = p_m + \rho g h + \frac{1}{2}\rho v_1^2,$$

where the velocity in the tube can be found from the continuity condition:

$$S_1 v_1 = S_2 u = S_2\sqrt{\omega^2 r^2 - 2gh}.$$

Therefore,

$$p_m = p_0 - \rho g h - \frac{1}{2}\rho\left(\omega^2 r^2 - 2gh\right)\left(\frac{S_2}{S_1}\right)^2.$$

Notice that the "boiling" starts when $p_m = p_k$. So,

$$\omega_m^2 r^2 = 2gh + \left(\frac{p_0 - p_k}{\rho} - gh\right)\left(\frac{S_1}{S_2}\right)^2;$$

finally we obtain

$$\omega_m = r^{-1}\sqrt{2gh + \left(\frac{p_0 - p_k}{\rho} - gh\right)\left(\frac{S_1}{S_2}\right)^2}.$$

4) The maximal productivity is apparently achieved for the highest efficiency. The efficiency is highest, when the residual velocity is lowest: $u \to 0$, and $\omega \to \omega_{\min}$. According to the results of the second question, $\omega_{\min} = r^{-1}\sqrt{2gh}$. So, the minimal residual velocity of the water streams is

$$v_{\min} = \omega_{\min} r = \sqrt{2gh}.$$

The associated lost power is $\frac{1}{2}\mu v_{\min}^2 = \mu g h$. The useful power is associated with the potential energy increase (by gh), i.e. the total power $P = 2\mu g h$. Hence,

$$\mu = \frac{P}{2gh}.$$