7. Heat exchange

1) It is easy to see that the temperature profile along the plate is linear, and the temperature difference ΔT between the two plates is constant, $\Delta T \equiv T_0 - T_2$. Indeed, then the heat exchange rate q (per unit plate area) is also constant, which in its turn corresponds to a linear temperature profile. Let us use a reference frame moving together with the incoming air. Then, the temperature increase rate at a given point is $\dot{T} = v(T_2 - T_1)/x$. Then, the heat balance for a air element of volume $V = s \times h$ is written as $\rho s h c_p \dot{T} = q = s\sigma\Delta T/d = s\sigma(T_0 - T_2)/d$. So, $\rho s h c_p v(T_2 - T_1)/x = s\sigma(T_0 - T_2)/d$, hence

$$T_2 = \frac{x\sigma T_0 + \rho h c_p v d T_1}{x\sigma + \rho h c_p v d}.$$

2) Rewriting the heat balance equation $P = -C\frac{dT}{dt}$ as $dt = -CP^{-1}\,dT$ we conclude that time can be found via the area S under the graph, where P^{-1} is plotted versus the temperature as $t = SC$. The graph data:

T(K)	100	200	300	400	500	600	700	800
P(W)	13	30	55	83	122	177	258	395
$100P^{-1}$	7.7	3.3	1.8	1.2	.82	.57	.39	.25

Substituting the region with smooth boundaries with a superposition of trapezoids we find $S \approx 12\,\mathrm{K/W}$. Consequently, $t = 120\,\mathrm{s}$.