5. Empty bag (12 p)

A cylindrical bag is made from a freely deformable fabric, impermeable for air, which has surface mass density σ; its perimeter L is much less than its length l. If this bag is filled with air, it resembles a sausage. The bag is laid on a horizontal smooth floor (coefficient of friction $\mu = 0$). The excess pressure inside the bag is p, the free fall acceleration — g. The density of air is negligible.

1) Find the width of the contact surface between the bag and the floor c.

2) Prove that the tension in the bag's fabric $T = \alpha x + \beta$, where x is the height of the given point P above the floor, and find the coefficient α. *Remark:* fabric's tension is (loosely speaking) the force per unit length. Actually, fabric's tension is not as simple concept as a rope's tension, because different force directions are possible; here, however, we neglect the force component along the "sausage's" axis. Thus, we can describe the fabric's tension here with a single number, the force per unit length T acting across an horizontal cut of the "sausage".

Hint: consider the force balance for a small piece of fabric (using the vertical cut shown in figure).

3) Let the highest point of the bag be at height a from the floor. What is the tension T_1 at this highest point? Express your answer in terms of a, σ and p (or α, if you were unable to answer the previous question).

Hint: consider the force balance between two halves of the bag.

4) Assuming that $p \gg \sigma g$, determine the quantity $\varepsilon = \frac{b-a}{b+a}$, where b is the width of the bag.

