7. Mass-spectrometer

1) The trajectory of a charged particle in the magnetic field is circle of radius $R = l/\sqrt{2}$. Lorenz force is responsible for the acceleration, $Bev = Mv^2/R$, hence $BeR = p$. Substituting $p^2 = 2MUe = B^2e^2R^2$, we obtain

$$M = \frac{B^2l^2e}{4U}.$$

2) Now, the radius can be $R \pm r$. Approximate calculus yields $\Delta R/R = r\sqrt{2}/l \approx \Delta M/2M$, hence $\Delta M \approx Mr2\sqrt{2}/l$, i.e.

$$\Delta M = \frac{B^2lre}{\sqrt{2}\,U}.$$

3) Ion leaves the magnetic field at the distance r before (or after) performing a quarter of the circle. So, $\Delta\varphi \approx r/R = r\sqrt{2}/l$.

4) Certain initial energy kT implies that the terminal energy $Ue + kT = e(U + kT/e)$; this is equivalent to the change of the voltage by $\delta U = kT/e$. Using approximate calculus and the result of the first question, we obtain: $\delta M = \frac{dM}{dU}\,kT/e$, i.e.

$$\delta M = \frac{B^2l^2kT}{4U^2}.$$