7. To the Mars (10 pts)

In this problem, we study a project for flying to Mars. At the first stage, the space ship switches on the rocket engines and obtains an initial velocity v_0. You may assume that during the first stage, the height of the space ship (from the surface of the Earth) remains much smaller than the radius of the Earth $R_0 = 6400\,\text{km}$. At the second stage, the space ship performs a ballistic motion in the gravity field of the Earth: upon achieving a height, which is much larger than the radius of the Earth R_0, but much smaller than the orbital radius of the Earth, $R_e = 1.5 \cdot 10^8\,\text{km}$.

1) Find the relationship between the residual velocity v_1 (with respect to the Earth) at the end of the second stage, and the quantities v_0, R_0, and the free fall acceleration at the surface of the Earth g (for the subsequent questions you may use the numeric value $g \approx 9.8\,\text{m/s}^2$).

At the third stage, the space ship performs a ballistic motion in the gravity field of the Sun, up to reaching an immediate neighborhood of the Mars. The trajectory is chosen by minimizing the residual launch velocity v_1 (required for achieving the Mars).

2) Sketch the trajectory.

3) Find the flight time T. You may use the following numeric data: the orbital velocity of the Earth $v_e = 30\,\text{km/s}$, orbital radius of the Mars $R_m = 2.3 \cdot 10^8\,\text{km}$.

4) Find the previously considered launch velocity v_0 and the terminal (i.e. at the end of the third stage) velocity v_t of the space ship with respect to the Mars.

5) The required mass of the fuel M can be found from the formula

$$v = u \ln[(M + m)/m],$$

where u is the speed of the gas at the outlet of the engine (with respect to the space ship), and m is the useful mass of the ship (when all the fuel is exhausted). You may assume $m \ll M$ and use the numerical value $u = 1\,\text{km/s}$. Find, how much more fuel is needed for the Mars flight, as compared to simply escaping the Earth gravity field, if the useful space ship mass is equal in both cases.