3. Temporal focusing (10 pts)

1) Apparently, $t_0 = (a+b)/v_0 = (a+b)\sqrt{m/2U_0e}$.

2) Apparently, $t = a/v_a + b/v_b \approx t_0 - [a(v_a - v_0) + b(v_b - v_0)]/v_0^2$. On the other hand, for constant voltage $U(t)$, $v_b = v_0$. For the interval between B and C, we have $\Delta(mv^2/2) \approx mv_0(v_a - v_0) = -Ue$. So,

$$t = t_0 + Uea/mv_0^3 = t_0 + \frac{U}{U_0}\frac{a}{2}\sqrt{\frac{m}{2U_0e}}\,.$$

3) For the changing voltage, we can use the result of the previous question, but the voltage value should be taken at the moment of electron passing the gap. Also, the terminal velocity is changed, $v_1 - v_0$ is related to the change of the kinetic energy $e[U(t+t_a) - U(t)]$. The travel time of the later electrons passing B at a certain moment of time $t > 0$ should as much shorter as it was delayed, i.e.

$$t_1 - t_2 = \sqrt{\frac{m}{8U_0^3e}}\,\{a[U(0) - U(t)] + b[U(t+t_a) - U(t)]\} = t,$$

where the flight time $t_a \approx a/v_0$ [since $U \ll U_0$]. Using the suggested approximation, we obtain

$$\sqrt{\frac{m}{8U_0^3e}}\,b[U(t + a/v_0) - U(t)]\} = t.$$

If we seek a quadratic solution $U(t) = At^2 + Bt$, we get

$$2At_a(t + t_a/2) + Bt_a = \frac{t}{b}\sqrt{\frac{8U_0^3e}{m}}.$$

So,

$$A = \sqrt{\frac{8U_0^3e}{m}}/2t_ab = \sqrt{\frac{8U_0^3e}{m}}\,\frac{\sqrt{2eU_0/m}}{ab} = \frac{4U_0^2e}{mab},$$

and $B = -At_a$. Finally we obtain

$$U(t) = \frac{4U_0^2e}{mab}\,t\left(t - a\sqrt{\frac{m}{2U_0e}}\right).$$

4) Lost are those electrons, which are in the interval between B and C, when the voltage drops to 0. The duration of the time interval, when these were emitted, is $\tau = a/v_0$. So, the asked fraction is given by

$$\tau/T = \frac{a}{T}\sqrt{\frac{m}{2U_0e}}\,.$$