7. To the Mars (10 pts)

1) Apparently $v_1^2 = v_0^2 - 2gR_0$.

2) The trajectory is ellipse touching Earth orbit at its perihelion and Mars orbit at its apohelion..

3) According to the Kepler's third law, the time $T = \frac{1}{2}T_0(a/R_e)^{3/2}$, where $T_0 = 1$ year is the period of Earth and $a = (R_e + R_m)/2$ is the longer semi-axes of the trajectory. Numerically, $T = 0,70$ years.

4) The full energy (kinetic plus potential) of the space ship is $E = -Gm/2a = -Gm/(R_e + R_m) = -Gm/R_e + \frac{m}{2}(v_1 + v_e)^2$. So,

$$\frac{1}{2}(v_1 + v_e)^2 = G\frac{R_m}{R_e(R_e + R_m)} = v_e^2\frac{R_m}{R_e + R_m}$$

and

$$v_1 = v_e\left(\sqrt{\frac{2R_m}{R_e + R_m}} - 1\right) \approx 0.095\, v_e \approx 2.86\,\text{km/s}.$$

Hence, $v_0 = \sqrt{11.2^2 + 2.86^2} = 11.6\,\text{km/s}$.

The relative speed near the Mars can be written using the appropriate change of indices in the expression for v_1 (alternatively, it can be found analogously to v_1):

we first find the speed with respect to Sun from the expression of the total energy: $v_3 =$

$$v_3 = v_m\left(\sqrt{\frac{2R_e}{R_e + R_m}} - 1\right) \approx -0.106\, v_e\sqrt{\frac{R_e}{R_m}} \approx -2.59\,\text{km/s}.$$

5) Apparently $M_i = me^{v_i/u}$, so that $M_a/M_b = e^{(v_a - v_b)/u} = e^{0.4} \approx 1.5$.