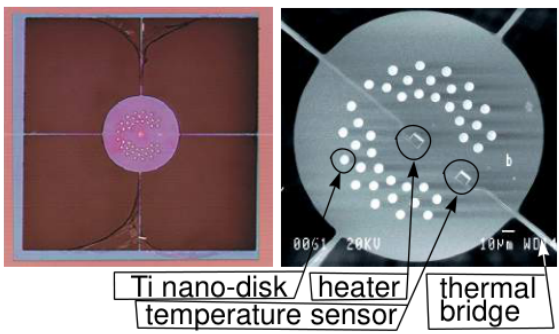
2. Microcalorimeter (9 points)

A microcalorimeter is a thin circular silicon nitride membrane, thermally isolated from the surroundings, except that it is thermally connected to the wafer by four thin and narrow thermal bridges (see Figure). The microcalorimeter is equipped with a small heater in the middle of the membrane and a similar structure on the edge of the membrane working as a thermometer. This micro calorimeter is used to study the thermal properties of nanoscale Ti disks (light tiny dots in Fig). The thermal power of the heater depends sinusoidally on time, $P = P_0 \cos(\omega t)$ (negative power implies a withdrawal of heat). The circular frequency ω is sufficiently low, so that for any moment of time t, the temperature of the microcalorimeter $T(t)$ can be considered constant across its entire surface, and the temperature profile along the thermal bridges can be considered linear. The wafer, to which the bridges are connected, is large and thick enough, so that its temperature T_0 can be considered to be constant all the time. Each of the four bridges have length L and cross sectional area of S; the thermal conductance of them is κ. Thermal conductance is defined as the heat flux (measured in Watts) per surface area, assuming that the temperature drop is 1 °C per 1 m. The heat capacity of the microcalorimeter (with Ti-disks) is C.



1) Find the thermal resistance R between the microcalorimeter and the wafer (i.e. the ratio of the temperature difference and heat flux).

For questions (ii) and (iii), use quantity R, without substituting it via the answer of question (i).

2) Write down the heat balance equation for the microcalorimeter and find the temperature of the microcalorimeter as a function of time $T(t)$ [you may seek it in the form $T = T_0 + \Delta T \sin(\omega t + \phi)$].

3) In order to study the thermal properties of the Ti-nanodisks, the amplitude of the sinusoidal oscillations of $T(t)$ should change by as large as possible value, as a response to a small change of C (which is caused by the Ti-disks). Find the optimal circular frequency ω_0.

4) We have assumed that the temperature profile along the bridges is linear, i.e. their heat capacity can be neglected. For high frequencies $\omega \gtrsim \omega_c$, this is not the case. Estimate the critical frequency ω_c in terms of κ, l, specific heat c and density ρ of the bridge material.