9. Asteroid (7 points)

It is believed that the impacts of the Earth with asteroids have played an important role in the history of Earth. In this problem, you are required to study such an impact. As an example, let us use the orbital data of the Apollo asteroid. Its perihelion is 0.65 AU, i.e. its closest distance from the Sun is $r_1 = \beta R$, where $\beta = 0.65$ and R denotes the radius of the Earth's orbit; its aphelion is 2.3 AU, i.e. its farthest distance from the Sun is $r_2 = \alpha R$, where $\alpha = 2.3$. In your calculations, you may use the orbital velocity of Earth, $v_0 = 30\,\mathrm{km/h}$, Earth radius $R_0 = 6400\,\mathrm{km}$, and the free fall acceleration at the Earth's surface, $g_0 = 9.8\,\mathrm{m/s^2}$. You may also use the formula $E = -GmM/2a$, expressing the total energy of a body of mass m, which moves along an elliptic orbit of longer semiaxis a, in the gravity field of a much heavier body of mass M. You may assume that the orbital planes of the Earth and of the asteroid coincide, and that they rotate in the same direction around the Sun.

1) Find the velocity v of the asteroid in the vicinity of the Earth, in the Sun's system of reference, neglecting the effect of Earth's attraction.

2) Find the radial and tangential components v_r and v_t of that velocity (i.e. the components, respectively parallel and perpendicular to the vector, drawn from the centre of the Sun to current position of the asteroid).

3) Find the same velocity components u_r and u_t in the Earth's system of reference.

4) Find the velocity of the asteroid w, immediately before entering the Earth's atmosphere (at the height $h = 100\,\mathrm{km}$ from the Earth's surface).