**2. Microcalorimeter** (9 points)

**1)** Every bridge has thermal resistance $L/\kappa S$; so, the overall resistance is $R = L/4\kappa S$.

**2)** The power dissipation $P$ results in an heat flux through the bridges, $\Phi = \Delta T/R$, and in the change of the heat contained in the microcalorimeter, $\dot{Q} = C\dot{T} = C\Delta\dot{T}$ (here, dot denotes the time derivative). So,

$$P_0 \cos(\omega t) = C\Delta\dot{T} + \Delta T/R.$$

Now, we can search the solution as $\Delta T = A\cos(\omega t+\phi)$, and denote $\psi = \arcsin\bigl(C\omega/\sqrt{C^2\omega^2 +$ Then,

$$P_0 \cos(\omega t) = A\sqrt{C^2\omega^2 + R^{-2}}\,\cos(\omega t + \phi - \psi).$$

So, we must have $\phi = \psi$ and $A = P_0/\sqrt{C^2\omega^2 + R^{-2}}$, i.e.

$$T = T_0 + \frac{P_0 \cos\left(\omega t + \arcsin\left(C\omega/\sqrt{C^2\omega^2 + R^{-2}}\right)\right)}{\sqrt{C^2\omega^2 + R^{-2}}}.$$

**3)** The amplitude of the oscillations is $A = \dfrac{P_0}{\sqrt{C^2\omega^2 + R^{-2}}}$; it must be as sensitive with respect to the small changes of $C$, i.e. $dA/dC$ must be maximal by modulus. $dA/dC = P_0(C^2\omega^2 + R^{-2})^{-3/2}C\omega^2$; if we denote $x = (C\omega)^2$, we need to minimize the following function of $x$:

$$\ln\bigl[(dC/dA)^2\bigr] = 3\ln(x + R^{-2}) - \ln x + \ln C$$

Upon taking derivative and putting it equal to 0, we obtain $3x = x + R^{-2}$, from where $x = R^{-2}/2$, i.e.

$$\omega = 1/\sqrt{2}\,CR.$$