**7. Fire** ***(6 points)***

The smoke will rise until its density becomes equal to the density of the air at the same height. Since the molar masses and pressures of the smoke and air are equal, this implies also equal temperatures (pressures are equal, because otherwise, there would be no mechanical equilibrium). Temperature of the smoke will drop with increasing height due to adiabatic expansion. If we combine the law of the adiabatic process $pV^\gamma = \text{Const}$ with the ideal gas law $(pV/T)^\gamma = \text{Const}$, we obtain $p^{\gamma-1}/T^\gamma = \text{Const}$. Taking a logarithm and differential from this equation, we obtain $(\gamma-1)\frac{dp}{p} - \gamma\frac{dT}{T} = 0$, hence we can use approximate expression for the require temperature change

$$20\,\mathrm{K} = \Delta T = T\,\frac{\gamma-1}{\gamma}\,\frac{\Delta p}{p}.$$

We can use $\Delta p = \rho g h$, where $\rho = p\mu/RT \approx 1.2\,\mathrm{kg/m^3}$ is the air density. Also, we can substitute $\gamma = c_p/c_V = (c_V + R)/c_V$. So, we obtain

$$\Delta T = \frac{R}{c_V + R}\,\frac{\mu g h}{R},$$

hence

$$h = \left(1 + \frac{c_V}{R}\right)\frac{\Delta T\, R}{\mu g} \approx 2040\,\mathrm{m}.$$