8. Electron (5 points)

Let us write the Newton's II law for x- and y-components of the electrons coordinates:

$$m\ddot{x} = -eE_0\omega\cos\omega t,$$
$$m\ddot{y} = eE_0\sin\omega t.$$

We can integrate these equations over time (bearing in mind that initial velocity is zero):

$$m\dot{x} = -eE_0\omega^{-1}\sin\omega t,$$
$$m\dot{y} = eE_0\omega^{-1}(1-\cos\omega t).$$

Now, we can integrate once more, bearing in mind that the initial coordinates are zero:

$$x = \frac{eE_0}{m\omega^2}(\cos\omega t - 1),$$
$$y = \frac{eE_0}{m\omega^2}\cos\omega t + \frac{eE_0}{\omega m}t.$$

So, the electron performs circular motion in the system of reference, moving with velocity (parallel to the y-axis) $u = \frac{eE_0}{\omega m}$. The radius of the orbit is $R = \frac{eE_0}{\omega^2 m}$. In the laboratory system, this is a cycloid (the curve drawn by a point on the edge of a rolling disk); the distance between the neighbouring loops is $\Delta = 2\pi u/\omega = 2\pi R$.