9. Asteroid (7 points)

1) The longer semiaxis is $a = \frac{1}{2}(r_1 + r_2) = \frac{1}{2}(\alpha + \beta)R$. So, the full energy of the asteroid at the Earth's location is

$$\frac{E}{m} = -\frac{\gamma M}{2a} = \frac{1}{2}v^2 - \frac{\gamma M}{R}.$$

Bearing in mind that $v_0^2 = \frac{\gamma M}{R}$, we can rewrite this as

$$\frac{v_0^2}{\alpha + \beta} = v_0^2 - \frac{1}{2}v^2.$$

So, we obtain

$$v = v_0\sqrt{2[1 - (\alpha + \beta)^{-1}]} \approx 34.5\,\text{km/h}.$$

2) Tangential component can be found from the angular momentum conservation law: $v_r = v_p\beta$, where the velocity at the perihelion can be found analogously to v:

$$v_p = v_0\sqrt{2\left(\frac{1}{\beta} - \frac{1}{\alpha + \beta}\right)} = v_0\sqrt{\frac{2\alpha}{\alpha + \beta}} \approx 37.5\,\text{km/h}.$$

So,

$$v_t = v_0\beta\sqrt{\frac{2\alpha}{\alpha + \beta}} \approx 24.4\,\text{km/h}.$$

Radial component

$$v_r = \sqrt{v^2 - v_t^2} \approx 24.4\,\text{km/h}.$$

3) The required components can be found by subtracting the Earth's orbital velocity. Apparently $u_r = v_r$, and

$$u_t = v_t - v_0 \approx -5.6\,\text{km/h}.$$

4) When the asteroid approaches Earth's surface along the parabolic orbit, the energy due to the Earth gravity force gR_0 is added to its kinetic energy in the Earth's system of reference:

$$w = \sqrt{u_t^2 + u_r^2 + 2gR_0} \approx 27.4\,\text{km/h}$$