2. Nanoclock ***(10 points)***

Nanotechnology allows controllable fabrication of very small structures. Let us consider a tiny homogeneously charged thin ring, having a radius R and carrying a positive net charge Q.

i) Find the electric potential φ at a point P, which is on the axes of the ring, at a distance z from the centre of the ring.

ii) Find the electric field E at the point P.

iii) Show that the force acting on an electron moving along the symmetry axes in the vicinity of the centre of the ring ($|z| \ll R$) is harmonic (i.e. depends linearly on z).

iv) Define the frequency of oscillations of such an electron. Use numerical values $R = 1\,\mu\text{m}$ and $Q = 1.0 \times 10^{-13}\,\text{C}$.

v) Now, let us assume that the electron can perform also off-axis movements. Is the position at the symmetry centre of the ring (on-axis, $z = 0$) stable or unstable? Motivate your answer.

Hint: You may use the approximate formula

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{1}{2}\alpha(\alpha-1)x^2.$$