10. Wedge ***(5 points)***

The center of mass of the system doesn't move, therefore $Mu = mv_h$, where u and v_h are the horisontal components of the velocities of the wedge and block in the lab frame at some instant of time. In the wedge's frame, the block has horisontal velocity $v' = v_h + u = v_h\left(1 + \frac{m}{M}\right)$. Since the block is sliding down the wedge, the vertical component of the block has to be $v_\downarrow = v' \tan\alpha$. The total kinetic energy in the lab frame $E_K = \frac{1}{2}\left(Mu^2 + mv_h^2 + mv_\downarrow^2\right)$. Substituting the relevant quantities and simplifying, we obtain

$$E_k = \frac{m}{2} v_\downarrow^2 \beta, \quad \text{where} \quad \beta \equiv \frac{M}{M+m}\cot^2\alpha + 1.$$

The kinetic energy equals to the change in potential energy: $\frac{m}{2}\beta v_\downarrow^2 = \Delta E_P = mg(h - h')$. By differentiating and noting that $dh = dt \cdot v_\downarrow$, we get $\beta\, dv_\downarrow = g\, dt$, $t = \frac{\beta}{g} v_{\downarrow\max}$. From the above written energy conservation law we get $v_{\downarrow\max} = \sqrt{\frac{2gh}{\beta}}$; so we finally have

$$t = \sqrt{\frac{2\beta h}{g}} = \sqrt{\frac{2h}{g}\left(\frac{M}{M+m}\cot^2\alpha + 1\right)}.$$