2. Nanoclock ***(10 points)***

i) All the charges of the ring are at the same distance from the point P, $l = \sqrt{R^2 + z^2}$. So, according to the superposition principle, the potential is the sum of potential of all the charges,

$$\varphi = kQ/\sqrt{R^2 + z^2}.$$

ii) $E = -\frac{d\varphi}{dz} = \frac{kQz}{(R^2+z^2)^{3/2}}$.

iii) For $|z| \ll R$ we may approximate $(R^2 + z^2)^{3/2} \approx R^3$, so that $E = \approx kQz/R^3$.

iv) From the Newton II law, $m\ddot{z} = -ekQz/R^3$, i.e. $\omega^2 = ekQ/mR^3$ and $f = \frac{1}{2\pi R}\sqrt{\frac{ekQ}{mR}} \approx 5.6 \times 10^{12}\,\mathrm{Hz}$.

v) At the origin, there are no charges; hence, the electric field lines cannot neither start nor end there. Consider a tiny coaxial cylinder embracing the origin. The field lines exit the cylinder through its bottom and top surfaces (because there is an electric field $E = \approx kQz/R^3$). Hence there must be field lines entering the cylinder through its side surface. This implies a radial repelling force for an electron situated at that surface, i.e. instability.

Remark: In the same way, one can prove a theorem, electrostatic equlibria are always unstable.