3. Ball ***(8 points)*** A straightforward way to solve parts **i)**, **ii)**, and **iii)** is to use conservation of angular momentum with respect to any axis laying on the surface (there is no torque with respect to these axes). Angular momentum at the beginning: $L_0 = mvr$. Rolling ball is always rotating around the touching point with the ground (although the location of that point is constantly changing). From Steiner's theorem we know that for a rotation axis that is located at the distance r form the center of mass, the moment of inertia is $I' = I + mr^2$. Therefore the angular momentum for rotating ball is $L' = I'\omega$ and clearly $L_0 = L'$, therefore $mvr = (I + mr^2)\omega$, $\omega = \frac{mvr}{I+mr^2}$.

i) The torque with respect to center of mass: $M = mg\mu r$. Since $M\Delta t = I\Delta\omega$, and the sliding stops when angular speed has reached value ω, we obtain the corresponding time: $t = \frac{I\omega}{mg\mu r} = \frac{Iv}{g\mu(I+mr^2)}$.

ii) Clearly, $\omega_* = \frac{mvr}{I+mr^2}$ and $E_* = \frac{1}{2}(mv_*^2 + I\omega_*^2) = \frac{1}{2}(mr^2\omega_*^2 + I\omega_*^2) = \frac{m^2v^2r^2}{2(I+mr^2)}$. If I increases then E_* decreases.

iii) Since we didn't make any assumptions about the functional form of μ while deriving w_*, the results w_* and E_* of **ii)** are still valid.

iv) The net force $F = F_\mu + F_r$ causes the decrease of translational velocity: $m\Delta v = F\Delta t$, $v'(t) = v - (\mu + \mu_r)gt$. The sliding stops when $\omega(t) = v'(t)/r$. Note that the torque with respect to center of mass is only caused by kinetic frictional force, therefore $\omega(t) = \frac{mg\mu r}{I}t$. We get an equation for the time of the termination of sliding: $\frac{mg\mu r^2}{I}t = v - (\mu + \mu_r)gt$, $t = \frac{Iv}{g\mu(I+mr^2)+g\mu_r I}$. The corresponding angular speed $\omega_*'' = \frac{mvr}{I+mr^2+\frac{\mu_r}{\mu}I}$, and energy $E_*'' = \frac{1}{2}(I + mr^2)\omega_*^2$. Clearly, $E_*'' < E_*$.