9. Stratostat *(5 points)*

i) Consider the pressure difference at heights $z+dz$ and z: $dp = -\rho g\, dz$ (the difference is simply due to the weight of the layer dz). The density can be found using the state equation of ideal gas: $pV = \frac{m}{\mu}RT$, $\rho = \frac{m}{V} = \frac{\mu p}{RT}$. Therefore $\frac{1}{p}\frac{dp}{dz} = -\frac{\mu g}{RT}$. Note that we have a derivative of a logarithm: $\frac{1}{p}\frac{dp}{dz} = \frac{d\ln p(z)}{dz}$. Therefore $\ln(p/p_0) = -\alpha z$, $p = p_0 e^{-\alpha z}$, where $\alpha = \frac{\mu g}{RT}$.

Alternatively, we can derive this law from the Boltzmann distribution for the particle density $n = n_0 e^{-U/kT}$, where U is the potential energy of a molecule. Bearing in mind that for a constant temperature, the pressure is propotional to the density, we obtain $p = p_0 e^{-U/kT}$. Substituting $U = mgz = \frac{\mu}{N_A}gz$ and $R = kN_A$, we obtain the same result as above.

ii) Clearly, the pressures inside and outside of the stratostat are equal and depending on the height as $p = p_0 e^{-\alpha z}$, where $\alpha = \frac{\mu_a g}{RT}$. For helium inside the sack $pV = \text{const}$. Let the volume of the stratostat be V_S, therefore $p_0 \beta V_S = pV_S$, where p is the pressure at the height, where helium has filled the entire volume. Since $p = \beta p_0$, we obtain $\frac{\mu_a g}{RT}h = -\ln\beta$, $h = \frac{RT}{\mu_a g}\ln\frac{1}{\beta}$, $h = 17\,\text{km}$.