Problem 11. Black box ***(10 points)*** There are several measurements, which can be made.

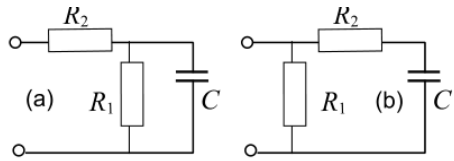
i. (2 pts) We can measure the voltage of the battery $\mathcal{E} \approx 3.2\,\mathrm{V}$.

ii. (2 pts) Then, we can connect battery to the outlets of the box via ammeter and measure the current. It appears that at the first moment, $I_{c0} \approx 1.3\,\mathrm{mA}$; however, the current starts to decrease (decreasing twice during $\tau_1 \approx 12\,\mathrm{s}$) and achieves at the long-time limit the final value $I_{c\infty} \approx 0.35\,\mathrm{mA}$.

iii. (2 pts) Further, we can measure voltage at the outlet after disconnecting the battery. At the first moment, $V_d \approx 2.35\,\mathrm{V}$; it decreases twice per $\tau_2 \approx 25\,\mathrm{s}$ and vanishes at the long-time limit.

iv. (4 pts) Finally, we can connect the ammeter to the outlet immediately after disconnecting the battery, and measure the current. Initially, it has value $I_d \approx 1.0\,\mathrm{mA}$, and vanishes at the long-time limit.

From iii and ii we can conclude that the box must contain a capacitor C (if there were an inductance, the current I_c would increase in time). Because of self-discharge (voltage vanishes for iii), there must be a resistance R_1 parallel to the capacitor. Because of a prolonged charging (for ii, $\tau_1 > 0$), there must be also a resistor R_2 in serial connection to the capacitor. So, there are two possible schemes, Fig (a) and Fig (b).



In case (a):

$$I_{c0} = \mathcal{E}/R_2, \qquad I_{c\infty} = \mathcal{E}/(R_1 + R_2),$$
$$I_d = \mathcal{E}R_1/R_2(R_1 + R_2), \qquad U_d = \mathcal{E}R_1/(R_1 + R_2).$$

In case (b),

$$I_{c0} = \mathcal{E}(R_2^{-1} + R_1^{-1}), \qquad I_{c\infty} = \mathcal{E}/R_1,$$
$$I_d = \mathcal{E}/R_2, \qquad U_d = \mathcal{E}R_1/(R_1 + R_2).$$

In both cases, we have two unknown quantities (R_1 and R_2), and four equations. It appears (follows from these equations) that in both cases, two equalities should hold between the measured quantities: $U_d = \mathcal{E}I_{c\infty}/I_{c0}$, and $I_{c0} = I_{c\infty} + I_d$. So, the effective (independent) number of equations is reduced by two, which still leaves two — just sufficient for finding R_1 and R_2, but not enough to distinguish between the cases (a) and (b). In fact, it can be shown that these two cases cannot be distinguished even if we study the time-dependences of voltage and currents. So, we can say that we have either scheme (a) with $R_2 = \mathcal{E}/I_{c0} \approx 2.5\,\mathrm{k\Omega}$ and $R_1 = \mathcal{E}/I_{c\infty} - R_2 \approx 6.9\,\mathrm{k\Omega}$, or scheme (b) with $R_1 = \mathcal{E}/I_{c\infty} \approx 9.1\,\mathrm{k\Omega}$ and $R_2 = \mathcal{E}/I_d \approx 3.2\,\mathrm{k\Omega}$.

The value of the capacitor can be estimated from characteristic current decay times. For instance, using the characteristic time τ_2, in the case (a) we have $\tau_2 = \ln 2\, R_1 C$, hence $C = \tau_2/\ln 2\, R_1 \approx 5.2\,\mathrm{mF}$. In the case (b), $\tau_2 = \ln 2(R_1 + R_2)C$, hence $C = \tau_2/\ln 2(R_1 + R_2) \approx 2.9\,\mathrm{mF}$.