Problem 1. Charges in E ***(8 points)***

i. (2.5 pts) The initial and final momentum of the dumbbell differ by $4mv$. The only external force acting on the dumbbell is the electrostatic force Eq, applied to the blue particle. So, the duration of that force must satisfy condition $Eq\tau = 4mv$, hence $\tau = 4mv/Eq$.

ii. (3 pts) Once the blue particle enters the electric field, the dumbbell's center of mass C obtains acceleration $a = Eq/2m$. Let us consider the motion in the system, where C is at rest. Red and blue particles move symmetrically in that system; let us consider the red particle. Due to the inertial force $F_i = ma = Eq/2$, the equilibrium position of the red particle is shifted (the half-spring is to be deformed by $x = F_i/2k = Eq/4k$ to achieve the equilibrium); the particle starts from rest, apart from the equilibrium. So, it starts oscillations, the circular frequency being given by $\omega = \sqrt{2k/m}$ (the factor 2 accounts for the fact that oscillations take place around the center of the spring and half-spring has twice larger stiffness). For the dumbbell to return with the same velocity as it approached, the residual oscillations must be absent (otherwise, some part of the initial kinetic energy would be turned into the oscillations energy, so that the center of mass velocity would be decreased). So, the oscillations phase needs to be $\omega\tau = 2n\pi$, where n is an integer. Since the spring's length achieves minimum only once, $n = 1$. So, $\omega\tau = 2\pi$, and the equality can be written as

$$\sqrt{\frac{2k}{m}}\,\frac{4mv}{Eq} = \pi.$$

iii. (2.5 pts) There is a requirement that the red particle never enters the region $x > 0$. The most critical moment is $t = \tau/2$ ($t = 0$ corresponds to the blue particle enetring the electric field), when the spring is maximally compressed. The center of mass has displaced by $s = at^2/2 = (Eq/2m) \cdot \tau^2/8 = mv^2/Eq$, and the spring half-length has decreased by $2x$ (x is the difference between lengths of the initial and equlibrium states; we need the differece between the lengths of the initial, i.e maximally stretched state, and maximally compressed state). So,

$$\frac{L}{2} > s + 2x = mv^2/Eq + Eq/2k.$$