Problem 3. Tyrannosaur (T. Rex) ***(6 points)***

i. (3 pts) Knowing that mass m is proportional to volume, the relationship between mass and length scale is $L = l(M/m)^{1/3}$. The force F on animal bones is proportional to its mass and to the cross-sectional area of the bone; hence, the area is propotional to the mass. So, $\frac{M}{m} = N$, from which $L = lN^{1/3} \approx 3.23\,\mathrm{m}$. The step length is half of the distance between two traces of the same leg, i.e. 2 m. This corrsponds to the angle between the legs $\alpha = 2\arcsin\frac{1}{3.2} \approx 36^\circ$, which seems reasonable.

ii. (3 pts) Let us model the leg with a physical pendulum. The leg can be approximated as a uniform rod attached from its upper end (the hip joint). Then its moment of inertia is $I = \frac{1}{3}\mathcal{M}L^2$, where $\mathcal{M}$ is the leg mass.

For small-angle swings of the pendulum, the only force acting on the leg is from the mass of the leg. Thus, the torque equation will be

$$\frac{1}{3}\mathcal{M}L^2 \cdot \ddot{\varphi} = -\mathcal{M}g\varphi\frac{L}{2};$$

here, φ is the angle of the leg, so that the gravity force's lever arm is equal to $\varphi\frac{L}{2}$. Hence, the circular frequency of the leg is $\omega = \sqrt{\frac{3g}{2L}}$. The displacement A corresponds to the whole period, i.e. the walking speed $v = A/T = A\omega/2\pi = \frac{A}{2\pi}\sqrt{\frac{3g}{2L}} \approx 1.2\,\mathrm{m/s} \approx 5\,\mathrm{km/h}$. So, the walking speed is comparable to that of a human.