Problem 8. Sprinkler ***(3 points)***

It is known that for a body thrown at some angle α to the horizon with a vleocity v, the maximal flight length is achieved with $\alpha = 45^\circ$ (this result can be also easily derived). That maximal flight length is found as $s_{\max} = vt/\sqrt{2}$, where the flight time t is obtained from the condition $gt = 2v/\sqrt{2}$. So, $s_{\max} = v^2/g$. This distance gives the radius of the circular region, watered by the sprinkler; its area is $S = \pi s_{\max}^2 = \pi v^4/g^2$.

i. (1.5 pts) Let us plot the flight distance s as a function of the angle α at the outlet of the sprinkler. This is a smooth curve with one maximum. For a range of distances from s to $s + \Delta s$, the amount of received water is (roughly speaking) proportional to the corresponding width of the angle range $\Delta\alpha$. So, the watering intensity is (roughly)propotional to $Q \propto \dfrac{\Delta\alpha}{\Delta s} = 1\Big/\dfrac{\Delta s}{\Delta\alpha}$. At the limit of small $\Delta\alpha$ and Δs, this transforms into a derivative: $Q \propto 1\Big/\dfrac{ds}{d\alpha}$, i.e. Q tends to infinity at the maximum of $s(\alpha)$. In other words, the best position is at the distance $s = s_{\max}$.