7. Vacuum bulb *(8 points)* Let us study how a vacuum can be created inside a bulb by pumping. Let the volume of the bulb be V, and the pump consist in a piston moving inside a cylinder of volume αV, where $\alpha \ll 1$. The pumping cycles starts with piston being pulled up; when the pressure inside the cylinder becomes smaller than inside the bulb, a valve V_A (connecting the cylinder and the bulb) opens and remains open as long as the piston moves up. When piston is released, it starts moving down, at that moment, the valve V_A closes. As long as the valve V_A is open, the pressures of the bulb and the cylinder can be considered as equal to each other. When the piston moves down, the pressure in the cylinder increases adiabatically until becoming equal to the outside pressure $p_0 = 10^5\,\mathrm{Pa}$; at that moment, another valve V_B opens leting the gas out of the cylinder. When the piston reaches the bottommost position, there is no residual air left inside the cylinder. Now, the piston is ready for being lift up: the valve V_B closes and V_A opens, marking the beginning of the next pumping cycle. The air inside the bulb can be considered isothermal, with the temperature being equal to the surrondin temperature T_0. The adiabatic exponent of air $\gamma = c_p/c_V = 1.4$.

i) (2 pts) How many pumping cycles N needs to be done to reduce the pressure inside bulb from $p = p_0$ down to $p = \beta p_0$, where $\beta \ll 1$?

ii) (2 pts) What is the net mechanical work done during such a pumping (covering all the N cycles)?

iii) (2 pts) What is the temperature of the air released from the cylinder to the surroundings at the end of the pumping process (when the pressure inside the bulb has become equal to βp_0)?

iv) (2 pts) According to the above described pumping scheme, there is a considerable loss of mechanical work during the period when the piston is released and moves down. Such a loss can be avoided if there is another pump, which moves in an opposite phase: the force due to outside air pressure pushing the piston down can be transmitted to the other pump for lifting the piston up. What is the net mechanical work done when such a pumping scheme is used?