2. Capacitor *(6 points)*

i) The energy is $W = CU^2/2 = \frac{1}{2}\varepsilon_0 \frac{A}{d} E^2 d^2 = \frac{1}{2}\varepsilon_0 A d E^2$; hence, the energy density $w = W/Ad = \frac{1}{2}\varepsilon_0 E^2$.

ii) There are two ways to calculate the force. First, we notice that the innermost charges q at the capacitor plates are affected by the electric field E, therefore there is a force qE acting upon these. The outermost charges, however, have no electric field around them (because outside the inter-plate space, there is no electric field). Due to the Gauss law, the electric field decreases linearly with the net charge left below the level of the current point (i.e. towards the inter-plate space). Therefore, the electric field averaged over the charges is just half of the maximal value E: $\langle E \rangle = \frac{1}{2}E$, and the net force acting on the plate is $F = Q\langle E \rangle = CEd\langle E \rangle = \frac{1}{2}\varepsilon_0 A E^2$.

The second way includes writing the energy balance for a small displacement of a plate: $F \cdot \delta d = \delta(Q^2/2C) = \frac{Q^2}{2\varepsilon_0 A}\delta d = \frac{1}{2}C^2E^2 d \cdot \delta d \Rightarrow F = \frac{1}{2}\varepsilon_0 A E^2$.

iii) Let us push away part of the water from the inter-plate space so that there will be a small region of plate area dA, where there is no water between the plates (here, Δp is the pressure difference between the inter-plate space and the outside regions). By doing so, we perform work $d \cdot \delta A \cdot \Delta p$, and increase the capacitor's energy:

$$\delta W = \delta(Q^2/2C) = \frac{Q^2 d}{2\varepsilon_0}\left[\frac{1}{\varepsilon A} - \frac{1}{\varepsilon(A - \delta A) + \delta A}\right].$$

So,

$$\delta W = \frac{Q^2 d(\varepsilon - 1)\cdot \delta A}{2\varepsilon_0 \varepsilon^2 A^2} = \frac{1}{2}\varepsilon_0 E^2 d(\varepsilon - 1)\cdot \delta A;$$

comparing this with the pressure work $d \cdot \delta A \cdot \Delta p$ we conclude that

$$\Delta p = \frac{1}{2}\varepsilon_0 E^2(\varepsilon - 1) \Rightarrow p = p_0 + \frac{1}{2}\varepsilon_0 E^2(\varepsilon - 1).$$