3. Charged cylinder ***(8 points)***

i) Moving surface charge creates a solenoidal surface current with the surface density $j = \sigma v = \sigma\omega r$. From the circulation theorem for a rectangular loop embracing a segment of surface current we obtain $\frac{Bl}{\mu_0} = jl$, where l is the length of the surface current segment (so that jl gives the current flowing through the loop). Hence, $B = \mu_0 j = \mu_0\sigma\omega r$.

ii) Using formula $\mathcal{E} = \frac{d\Phi}{dt} = B\frac{dS}{dt}$, where S is the area covered by the wire, we obtain $\mathcal{E} = B\omega r^2/2$. Indeed, during a small time interval dt, the wire covers a equilateral triangle of side lengths r, r, and $r\omega dt$; its area is apparently $r^2\omega dt/2$. By using the earlier obtained expression for B we end up with

$$\mathcal{E} = \mu_0\sigma\omega^2 r^3/2.$$

iii) We need to show that from the previous task, $\frac{dS}{dt}$ is independent of the wire shape. First we note that due to rotational symmetry, $\frac{dS}{dt}$, it cannot depend on the rotation angle, i.e. $\frac{dS}{dt} \equiv \dot{S} = \text{Const}$. Further we note that regardless of the wire shape, during the entire rotation period $2\pi/\omega$, the whole circle area is covered; $\dot{S} \cdot 2\pi/\omega = \pi r^2 \Rightarrow \dot{S} = r^2\omega/2$.