6. Violin string ***(9 points)***

i) When the plate slides, there is a constant friction force $\mu_2 N$ acting upon the block, which means that the equilibrium deformation of the spring is $x_0 = \mu_2 N/k$; the net force acting upon the block (due to spring and friction) is given by $F = -k\xi$, where we have defined $\xi = x - x_0$. Therefore, while sliding, the block oscillates harmonically around the point $\xi = 0$. Slipping starts when the static friction will be unable to keep equilibrium, i.e. at $kx = \mu_1 N$, which corresponds to $\xi_0 = (\mu_1 - \mu_2)N/k$. If the plate moves slowly, the block is released with essentially missing kinetic energy, and the energy conservation law yields

$$\frac{1}{2}k\xi_0^2 = \frac{1}{2}mv_{\max}^2 \Rightarrow v_{\max} = \xi_0\sqrt{k/m}.$$

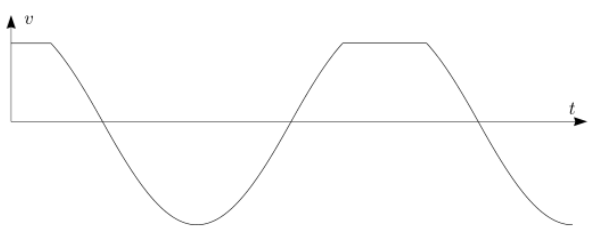
ii) As mentioned, when the plate slides, the motion of the block is harmonic, i.e. the graph of $x(t)$ is a segment of a sinusoid; when there is no sliding, the block moves together with the plate, i.e. the graph of $x(t)$ is a straight line. At the moment when slipping starts or stops, the oscillatory speed is equal to the speed of plate, i.e. the straight line is tangent to the sinusoid. The length of a straight segment can be calculated as

$$T_1 = 2\xi_0/u = 2(\mu_1 - \mu_2)N/ku;$$

the sinusoidal segment corresponds to a half-period and therefore has a length of $T_2 = \pi\sqrt{m/k}$.



iii) The speed $v(t) = \frac{dx}{dt}$ is the derivative of $x(t)$; therefore, the sinusoidal segment of $x(t)$ will correspond to a sinusoidal segment of $v(t)$, and a straight segment of $x(t)$ — to a horizontal segment of $v(t)$. The resulting graph is depicted below.



iv) Let the amplitude of the oscillations be A, i.e. the sinusoidal segments follow the law $\xi(t) = A\cos(\omega t)$, where $\omega = \sqrt{k/m}$. Correspondingly, $v(t) = A\omega\sin(\omega t) \Rightarrow A\sin(\omega t) = v(t)/\omega$; hence, for any point at a sinusoidal segment, $\xi^2 + v^2/\omega^2 = A^2$. At a point, where a sinusoid and a straight line meet, the straight line and sinusoid have equal values for $\xi = \xi_0 = (\mu_1 - \mu_2)N/k$ and $v = u$. Consequently,

$$(\mu_1 - \mu_2)^2 N^2/k^2 + u^2/\omega^2 = A^2 \Rightarrow A = \frac{1}{k}\sqrt{(\mu_1 - \mu_2)^2 N^2 + u^2 mk}.$$

v) The oscillations will be almost harmonic when the straight segments are very short, i.e. when $u/\omega \gg (\mu_1 - \mu_2)N/k \Rightarrow u \gg (\mu_1 - \mu_2)N/\sqrt{mk}$.