Problem 1. Asteroid ***(14 points)***

Consider a hypothetical asteroid of mass m_a and radius r_a, which moves along an elliptical orbit around the Sun (of mass M_s) in the same direction as Earth. Let us assume that the Earth's orbit is a circle of radius R_e (neglecting thus its eccentricity) and that the two orbits lay in the same plane. The asteroid's shortest distance to the Sun (at its perihelion) $R_{\min} = \frac{1}{2}R_e$ and the longest distance (at the apohelion) $R_{\max} = 1.51R_e \approx 1.5R_e$ (you may use the approximate value to simplify your calculations). The orbital velocity of the Earth $v_0 = 30\,\mathrm{km/s}$. You can also use the following numerical values: the radius of the Earth $r_e = 6400\,\mathrm{km}$, free fall acceleration at the Earth's surface $g = 9.81\,\mathrm{m/s^2}$, angular diameter of the Sun as seen from the Earth $\alpha = 0.5^\circ$, duration of one year $T_0 = 365$ days, the temperature of the Sun's surface $T_s = 6000\,\mathrm{K}$, free fall acceleration at the Sun's surface $g_s = 275\,\mathrm{m/s^2}$, Stefan Boltzmann constant $\sigma = 5.6704 \times 10^{-8}\,\mathrm{kg \cdot s^{-3} \cdot K^{-4}}$, the speed of light $c = 3 \times 10^8\,\mathrm{m/s}$. The asteroid is of a spherical shape, its radius $r_a = 10\,\mathrm{m}$ and mass $m_a = 1 \times 10^7\,\mathrm{kg}$; both the Sun and the asteroid can be considered as perfectly black bodies.

Part A. Collision with Earth ***(5 points)***

i. (2 pts) Suppose that the asteroid will collide with the Earth and is already very close, at a distance $l \ll R_e$ from the Earth's surface; what is the velocity of the asteroid with respect to the Earth assuming that (a) $l \gg r_e$; (b) $l \ll r_e$.

ii. (2 pts) Impact parameter b is defined in the Earth's reference frame as the distance between the Earth and a line, tangent to the asteroid's trajectory at a point which is far enough (at a distance l, $r_e \ll l \ll R_e$). Determine the maximal value of the impact parameter $b_{\max}$ for which the asteroid will still collide with the Earth.

iii. (1 pt) According to calculations, the asteroid is going to hit the Earth centrally after $N = 10$ orbital periods. In order to avert the collision, the period of the asteroid needs to be changed; by how many seconds? (Assume simplifyingly that the intersection point of the two orbits remains at rest.)

Part B. Changing the solar pull ***(9 points)***

Theoretically, it is possible to change the asteroid's period by making use of the pressure of the solar radiation. Let us study, how realistic is such a project. The Sun pulls the asteroid with the gravitational force equal to $F_0 = GM_s m_a/R^2$, where G is the gravitational constant and R is the distance between the Sun and the asteroid at the given moment of time. Let us denote $GM_s = \gamma_0$, so that

$$F = \gamma_0 m_a / R^2.$$

Suppose that when the asteroid is at its perihelion, the constant γ_0 is decreased instantaneously, down to a new value γ_1, which remains constant during the subsequent motion.

i. (2 pts) Find the new apohelion distance $R'_{\max}$ of the asteroid; express it in terms of $\kappa = (\gamma_0 - \gamma_1)/\gamma_0$.

ii. (2 pts) Find the change of the asteroid's orbital period assuming that $\kappa \ll 1$ (express it in terms of κ).

iii. (4 pts) Suppose that at its perihelion, the asteroid is coated with a perfectly retro-reflective paint (which directs all the incident light directly back, towards the source). Such a painting will result in a change of the effective pull of the asteroid towards the Sun; find the respective value of κ (provide also a numerical estimate).

iv. (1 pt) Estimate, is it realistic to avert the collision with this asteroid using the retroreflective paint.