Problem 6. Uranium decay ***(7 points)***

Natural uranium consists of 99.3% U^{238}, and in this problem we can ignore the presence of other isotopes. U^{238} decays according to the table at the bottom of the page, where an isotope decays into the isotope indicated in its neighbouring cell (to the right), the decay energy is given in the cell below in megaelectronvolts (MeV), $1\,\mathrm{MeV} = 1.6 \times 10^{-13}\,\mathrm{J}$, and the last row indicates the logarithm of the respective half-life in seconds, $\log_{10}\tau_{1/2}$ (so that 17.15 below U^{238} means that the half-life of U^{238} is $10^{17.15}\,\mathrm{s} \approx 1.41 \times 10^{17}\,\mathrm{s}$).

You may also use the value of the Avogadro number $N_A = 6.02 \times 10^{23}\,\mathrm{mol}^{-1}$, and the following physical properties of uranium: melting point $T_0 = 1408\,\mathrm{K}$, density $\rho = 1.89 \times 10^4\,\mathrm{kg/m^3}$, thermal conductivity $\kappa = 27.5\,\mathrm{W/m \cdot K}$, molar mass $\mu = 0.238\,\mathrm{kg/mol}$. It can be assumed that the radon which appears in the chain of nuclear decays does not escape the bulk of uranium. *Remark:* thermal conductivity is the coefficient of proportionality between the heat flux density ($\mathrm{W/m^2}$) and dT/dx (temperature drop per unit distance).

i. (2 pts) Assuming that the only source of natural U^{234} is the decay chain of U^{238}, what is the percentage of U^{234} in the natural uranium ore?

ii. (2 pts) Determine the heat production volume rate of natural uranium w (in watts per cubic meter) due to nuclear decay.

iii. (3 pts) If the radius of a uranium ball is large enough, it will melt inside. How large radius R_0 is needed for such a melting, if ambient temperature is $T_a = 300\,\mathrm{K}$?

U^{238}	Th^{234}	Pa^{234}	U^{234}	Th^{230}	Ra^{226}	Rn^{222}	Po^{218}	Pb^{214}	Bi^{214}	Po^{214}	Pb^{210}	Bi^{210}	Po^{210}	
4.27	0.27	2.27	4.86	4.77	4.87	5.59	6.12	1.02	3.27	7.88	0.06	1.43	5.41	
17.15	6.32	4.38	12.89	12.38	10.70	5.52	2.27	3.21	3.08	−3.78	8.85	5.64	7.08	s