Problem 6. Uranium decay ***(7 points)***

i. (2 pts) It can be seen from the table that the fist half-life is much longer than all the others. This means that as soon as something is produced by the decay of U^{238}, all the other decay steps in the chain take place almost immediately, and for the other isotopes, a quasi-stationary concentration level is achieved — such that the number of decays per unit time of the isotope equals to that of U^{238}. Let us apply this to U^{234}. If the number of U^{238} atoms is N_{238} then the number of decays per unit time is $\frac{dN}{dt} = N_0 \ln 2/\tau_{238} = N_{234} \ln 2/\tau_{234}$, hence $N_{234}/N_{238} = \tau_{234}/\tau_{238}$ The total number of uranium atoms equals to $N = N_{238}/0.993$, so

$$\frac{N_{234}}{N} = \frac{\tau_{234}}{0{,}993\,\tau_{238}} \approx 5.53 \times 10^{-5}.$$

ii. (2 pts) Since the uranium ore has reached a quasi-stationary composition of isotopes, per each decay of U^{238}, there is one decay event for each of the isotopes. So we need to sum up all the decay energies in the second row of the table, this gives us $E_{\text{dec}} = 52.1\,\text{MeV}$. Then the heat production rate is given by $w = N_A \frac{\rho}{\mu} E_{\text{dec}} \frac{\ln 2}{\tau_{238}} \approx 2.0\,\text{W/m}^3$.

iii. (3 pts) The heat released will escape owing to the thermal conductance. Inside a sphere of radius r, the heat released equals to $\frac{4}{3}\pi w r^3 = 4\pi r^2 \kappa \frac{dT}{dr}$ (the right-hand-side gives the thermal flux due to conductance). From this equation we obtain $r\,dr = 3\frac{\kappa}{w}\,dT$, which yields after integration

$$R = \sqrt{6\frac{\kappa}{w}(T_0 - T_a)} \approx 305\,\text{m}.$$