Problem 7. Lifting by current ***(7 points)***

i. (2 pts) The Ampère force pulls the wires to the side so that the wires will take a curved shape. Since the Ampère force is perpendicular to the wire, the mechanical tension is constant along the wires. Let the tension be T and the curvature of the wire at a certain point — R. Let us consider a short piece of the wire, of length $a \ll R$. Then the angle by which the tangent of the wire rotates while the tangent point moves over an arc of length a is given by $\alpha = a/R$. Let us study the force balance in the perpendicular direction for that piece of wire: the Ampère's force IaB is balanced by the tension $T\alpha = Ta/R$. So, $R = T/IB$ which means that R is constant, and the wire will take the form of a circle segment. To conclude, both halves of the wire will take the form of a circle segment, the convex sides of which are turned outside.

ii. (2 pts) The maximal height is achieved when the circle segments form a perfect circle, in which case the lifting height is $\Delta h = l(1 - \frac{2}{\pi})$.

iii. (2 pts) If the central angle of the circle segments is 2α, the tangents to the wires at the point where the load is fixed forms angle α with the vertical direction. So the lifting force is $mg = 2T\cos\alpha$. From the other hand, $R = l/2\alpha = T/IB$, ie.

$$\alpha \frac{mg}{lIB} = \cos\alpha,$$

which is the equation from where one can determine the angle α. Then, the lifting height

$$\Delta h = l - 2R\sin\alpha = l - \frac{2T}{lIB}\sin\alpha = l\left(1 - \frac{\sin\alpha}{\alpha}\right).$$

iv. (1 pt) From the previous result it can be seen that we need to have $\frac{\sin\alpha}{\alpha} = \frac{\pi}{3}$, hence $\alpha = \frac{\pi}{6}$ and

$$I = \frac{mg\alpha}{lB\cos\alpha} = \frac{mg\pi}{3\sqrt{3}lB}.$$