Problem 8. Elastic collision ***(7 points)***

i. (1 pt) The centre of mass moves with the velocity $\vec{u} = \frac{M}{M+m}\vec{v}$, and that will be the speed of the small ball in new reference frame, hence its momentum $\vec{p} = -m\vec{u} = -\frac{Mm}{M+m}\vec{v}$. Since in this frame, the centre of mass is at rest, the large ball needs to have equal by modulus and opposite momentum.

ii. (3 pts) Let the balls change a momentum $\vec{q}$. The small ball will have momentum $\vec{p}' = \vec{p}+\vec{q}$, and as the centre of mass remains at rest, the large ball will have momentum $-\vec{p}'$. The energy conservation law can be written now as follows:

$$\frac{\vec{p}^{\,2}}{2m} + \frac{\vec{p}^{\,2}}{2M} = \frac{\vec{p}^{\,\prime 2}}{2m} + \frac{\vec{p}^{\,\prime 2}}{2M} \Rightarrow |\vec{p}| = |\vec{p}^{\,\prime}|,$$

ie. the moduli of the momenta will remain unchanged.

iii. (3 pts) In the laboratory frame, the momentum of the large ball will be

$$\vec{p}^{\,\prime\prime} = M\vec{u} - \vec{p}^{\,\prime};$$

since $|\vec{p}^{\,\prime}|$ remains constant, the angle α between $\vec{p}^{\,\prime\prime}$ and $\vec{u}$ will be maximal when $\vec{p}^{\,\prime\prime} \perp \vec{p}^{\,\prime}$, with

$$\alpha = \arcsin \frac{|\vec{p}^{\,\prime}|}{|M\vec{u}|} = \arcsin \frac{m}{M}.$$