Problem 9. Power lines ***(7 points)***

i. (2 pts) Let us use a frame where the moving perturbations are at rest. There, the centripetal acceleration required for the motion along a trajectory of curvature radius R is given by the mechanical tension of the rope, for a small piece of rope (of length l)

$$T\frac{l}{R} = \frac{v^2}{R}\sigma l \Rightarrow v = \sqrt{\frac{T}{\sigma}},$$

ie. $k = 1$.

ii. (2 pts) We consider the torque balance for one half of the wire, with respect to the point where it is fixed to the pole. Then, the centre of mass lays approximately at the distance $\frac{L}{4}$ (since the shape of the wire is not far from a straight line), and the equation can be written as

$$Td = \frac{L}{4}\sigma g\frac{L}{2} \Rightarrow T = \frac{L^2\sigma g}{8d}.$$

iii. (3 pts) For the natural oscillation modes, there will be standing waves with nodes at the fixing points; the lowest frequency corresponds to the longest wavelength, which is $2L$, so that $f_0 = v/2L = \frac{1}{2L}\sqrt{\frac{T}{\sigma}}$.