4. MAGNETIC DIPOLES (7 points)

Let us consider the following model for a magnetic dipole. Some wire with no resistance has been bent into the shape of a square with side length a. At some point on the wire is a small ideal current source that keeps current I flowing in the circuit in all situations. The magnetic moment m of a planar circuit is given by the relation $m = IA$, with vector $\vec{m}$ pointing in the normal direction of the circuit according to the right hand rule (A is the area bounded by the circuit).

i) *(3 points)* The dipole is placed inside a homogeneous magnetic field $\vec{B}$, so that the angle between $\vec{m}$ and $\vec{B}$ is θ. Find the angles θ_s and θ_u that correspond to stable and unstable equilibria, respectively. Calculate the amount of work (w) needed to rotate the dipole from θ_s to θ_u. Give your answer in terms of m and B.

We can use this model to calculate the magnetic properties of materials containing unpaired electrons that have negligibly weak interactions with one another. Let us consider a sample of material with n such unpaired electrons per unit volume, placed inside a homogeneous magnetic field $\vec{B}$. Due to spin, each unpaired electron acts as a small magnetic dipole. However, owing to the quantum nature the electron, the projection of its magnetic moment along $\vec{B}$ can only be μ_B or $-\mu_B$ (μ_B is called the Bohr magneton).

ii) *(4 points)* Calculate M, the magnetic moment per unit volume of the sample, if the temperature of the material is T and the external magnetic field is B.