8. ZENER DIODE (7 points) An inductance L and a capacitor C are connected in series with a switch. Initially the switch is open and the capacitor is given a charge q_0. Now the switch is closed.

i) *(1 point)* What are the charge q on the capacitor and the current I in the circuit as functions of time? Draw the phase diagram of the system — the evolution of the system on a $I - q$ graph — and note the curve's parameters. Note the direction of the system's evolution with arrow(s).

A *Zener diode* is a non-linear circuit element that acts as a bi-directional diode: it allows the current to flow in the positive direction when a forward voltage on it exceeds a certain threshold value, but it also allows a current to flow in the opposite direction when exposed to sufficiently large negative voltage. Normally the two voltage scales are quite different, but for our purposes we will take a Zener diode with the following volt-ampere characteristics: for forward currents, the voltage on the diode is V_d, for reverse currents, the voltage on the diode is $-V_d$, for zero current the voltage on the diode is $-V_d < V < V_d$.

Now we connect the inductance L, the capacitor C all in series with a switch and a Zener diode. The switch is initially open. The capacitor is again given the charge $q_0 > CV_d$ and the switch is then closed.

ii) *(2 points)* Make a drawing of the phase diagram for the system. Note the direction of the system's evolution with arrow(s).

iii) *(2 points)* Does the evolution of the system only necessarily stop for $q = 0$? Find the range of values of q on the capacitor for which the evolution of the system will necessarily come to a halt.

iv) *(2 points)* Find the decrease Δq in the maximum positive value of the capacitor's charge q after one full oscillation. How long does it take before oscillation halts?