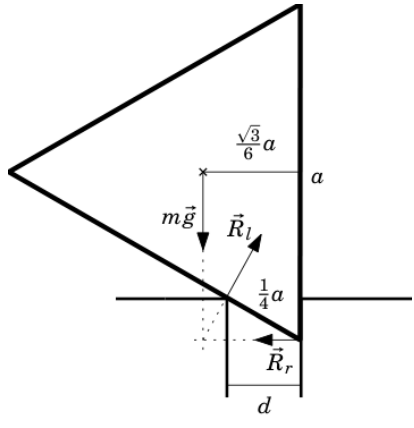
1. PRISM (8 points)

i) *(4 points)* The prism is acted on by three forces: reaction force $\vec{R}_l$ from the left-hand table, directed perpendicularly to the prism's face; reaction force $\vec{R}_r$ from the right-hand table, directed horizontally (with its point of action to be determined yet); and gravitational force mg, directed vertically and applied at the triangle's centre. (Considering a planar triangular cross-section of the prism is enough.)

On the verge of falling out, the force $\vec{R}_r$ is applied at the lower corner of the triangle. If a body in equilibrium is acted on by three forces, then their lines of action must intersect at one point. This is because otherwise the torque of one of those forces would not be zero with respect to the intersection of the lines of action of the two other forces.

As the distance between the triangle's centre and its side is $\frac{\sqrt{3}}{6}a$, the distance between the points of action of $\vec{R}_l$ and $\vec{R}_r$ is $\frac{\sqrt{3}}{6}a\cos 30° = \frac{1}{4}a$. Thus, $d = \frac{1}{4}a\cos 30° = \frac{\sqrt{3}}{8}a$.



ii) *(4 points)* Let the corners be denoted by A and B, and the tip of the prism (at its equilibrium position) by C. Let us consider a small rotation of the prism (assuming it remains in contact with the corners). The trajectory of the tip is a circle ascribed around the triangle ABC (it follows from the property of the inscribed angles because the $\angle ACB$ remains equal to 60°). The radius of that circle $r = l/\sqrt{3}$; its centre will be denoted by O. Once the prism rotates by angle α, so that the new position of the tip will be D, the central angle $\angle COD = 2\alpha$. Hence, the tip is raised by $r - r\cos(2\alpha) \approx 2r\alpha^2$. The height of the centre of mass P of the prism is raised because the tip is raised, and lowered because the vertical projection of the segment CP is reduced by $|CP|(1-\cos\alpha) \approx |CP|\alpha^2/2$. Here, $|CP| = a/\sqrt{3}$. So, the original position is stable if $\frac{a}{\sqrt{3}}\alpha^2/2 < 2\frac{l}{\sqrt{3}}\alpha^2$, hence $l > \frac{1}{4}a$.

