2. CELLPHONE CAMERA (6 points) The distance L is often called hyperfocal distance in photography and it was calculated more than one hundred years ago by Louis Derr (the figure is taken from his book *Photography for students of physics and chemistry*, published in 1906).

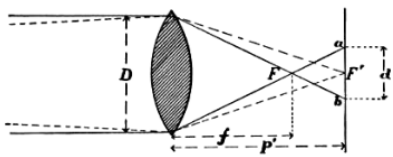


FIG. 49. Hyperfocal Distance.

Let's consider that the camera is focused to distance L and the image is formed exactly on the sensor's plane. The object's distance L and its image's distance a (corresponds to p' on the figure) are related by the lens formula $\frac{1}{L} + \frac{1}{a} = \frac{1}{f}$, thus

$$a = \frac{Lf}{L-f} = \frac{Lf}{L(1-f/L)} \approx \frac{Lf}{L}\left(1 + \frac{f}{L}\right) = f + \frac{f^2}{L},$$

where the approximation $(1+x)^{-1} \approx 1 - x$ (for small x) was used. Image's distance exceeds the focal length by $\Delta a = a - f = f^2/L$.

i) *(4 points)* The light coming from an infinitely far away object will pass the focal point F and form a cone which is cut by the sensor's plane. The diameter d of the cut on the sensor's plane can be found from similar triangles $d/D = \Delta a/f$, thus $d = Df/L$. Taking into account the sharpness condition $d \leq \eta$, where $\eta = w/N$ is the size of a single element of the sensor, we find that the limiting value of L is $L = Df/\eta = DfN/w \approx 5.5\,\mathrm{m}$.

ii) *(2 points)* We'll now find the shortest distance s satisfying the sharpness condition. Object at distance s will have an image at distance $b = f + f^2/s$ and the light passing the lens will converge behind the sensor's plane forming a cone. The diameter d_2 of the cone's cut with the sensor's plane can be calculated from similar triangles: $d_2/D = (b-a)/b$. Accounting for sharpness condition $d_2 = \eta$, we can express $b = a/(1-\eta/D)$, and substituting the values of a and b gives

$$f + \frac{f^2}{s} = \frac{f + \frac{f^2}{L}}{1-\eta/D} = f\frac{1+\eta/D}{1-\eta/D} \approx f\left(1 + \frac{\eta}{D}\right)^2 \approx f\left(1 + \frac{2\eta}{D}\right).$$

Finally, $f^2/s = 2f\eta/D$, or $s = \frac{1}{2}Df/\eta = L/2 \approx 2.75\,\mathrm{m}$.