3. MISSION TO MARS (7 points)

i) *(1 point)* We can find the orbital period of Mars from Kepler's third law $R_a^3/R_g^3 = T_a^2/T_g^2$, giving $t_t \approx 1.87\,\mathrm{yr}$.

ii) *(1.5 points)* Again, we can use Kepler's third law to calculate half of the orbital period.

$$t_t = \frac{T_g}{2}\frac{(R_a + R_g)^{3/2}}{(2R_g)^{3/2}} \approx 0.707\,\mathrm{yr}.$$

iii) *(1.5 points) Background.* Δv is important, because the sum of all Δv determines how much fuel is needed for a given mission. The fuel needed is exponential of total Δv and is described by Tsiolkovsky rocket equation.

Kinetic energy per unit mass of such a transfer orbit where it intersects the Earth's orbit is $-\frac{GM_s}{R_g+R_a} + \frac{GM_s}{R_g}$. Using the orbital angular speed of Earth we can substitute $GM_s = \frac{4\pi^2 R_g^3}{T_g^2}$. The speed at the beginning of the transfer orbit becomes

$$v_{t_0} = \sqrt{2GM_s\left(\frac{1}{R_g} - \frac{1}{R_g + R_a}\right)} \approx 32.7\,\mathrm{km/s}.$$

The speed in Earth's inertial frame is $v'_{t_0} = v_{t_0} - v_g \approx 2.94\,\mathrm{km/h}$. To achive that, we first need to escape Earth's gravity, so

$$\Delta v_1 = \sqrt{2\left(\frac{v'^2_{t_0}}{2} + \frac{GM_g}{r_g}\right)}.$$

Using the surface gravity of Earth we can substitute $\frac{GM_g}{r_g} = g_g r_g$ so $\Delta v_1 \approx 11.2\,\mathrm{km/s}$.

iv) *(1.5 points)* We can calculate the speed of the transfer orbit where it intersects the orbit of Mars from Kepler's second law $v_{t_1} = v_{t_0}/1.52 \approx 21.5\,\mathrm{km/s}$. The speed of the spacecraft relative to Mars is $v'_{t_1} \approx 3.25\,\mathrm{km/s}$. The speed of the spacecraft once near Mars surface is

$$v_{t_m} = \sqrt{2\left(\frac{v'^2_{t_1}}{2} + r_a g_a\right)} \approx 5.98\,\mathrm{km/s}$$

Since the speed of low Mars orbit is $v_{e_a} = \sqrt{r_a g_a} \approx 3.55\,\mathrm{km/s}$, we need to brake for $\Delta v_2 \approx 2.43\,\mathrm{km/s}$.

v) *(1.5 points)* The Earth–Sun–Mars angle α at the launch of the mission needs to be $\alpha = \pi - w_a t_t \approx 0.77$ for the spacecraft to reach Mars. Likewise for the return trip $\beta = \pi - w_g t_t \approx -1.301$. If we go to the corotating frame of referense with earth, we can see that the minimal time between those two angles is $\frac{2\pi-\alpha+\beta}{w_a - w_g} \approx 1.96\,\mathrm{yr}$. The minimal duration of the trip is therefore longer by t_t, giving 2.67 yr.