7. BURNING WITH A LENS (7 points) The solar energy flux which is focused by the lens to the image of the Sun can be calculated as $P = \frac{\pi}{4} d^2 I$; the image of the Sun radiates according to the Stefan-Boltzmann law with the total power $P = \frac{\pi}{4}(\alpha f)^2 \sigma T^4$. From the heat balance we obtain $\frac{\pi}{4} d^2 I = \frac{\pi}{4}(\alpha f)^2 \sigma T^4$, hence

$$T = \sqrt{\frac{d}{\alpha f}\sqrt{\frac{I}{\sigma}}} \approx 4500\,\mathrm{K}.$$

Due to the second law of thermodynamics, it is impossible to direct heat energy from a lower temperature body to a higher temperature body. Hence, the image temperature cannot exceed the temperature of the Sun. Now we can use the known temperature of Sun $T_0 = 6000\,\mathrm{K}$, but it is better to use the Stefan-Boltzmann law for solar radiation flux density: near the Sun's surface, $I_0 = \sigma T_0^4$, with the total flux of $P_t = 4\pi R_s^2 I_0$. Near the Earth, the total flux $P_t = 4\pi L^2 I$; here, R_s is the Sun's radius, and L — the orbital radius of the Earth. From here we obtain $I = I_0 R_s^2 / L^2 = \sigma T_0^4 R_s^2 / L^2$; using the previous result,

$$T = T_0 \sqrt{\frac{d}{\alpha f}\frac{R_s}{L}}.$$

Let us note that $\alpha L = 2R_s$, hence

$$T = T_0 \sqrt{\frac{d}{2f}} \le T_0,$$

which means that $d \le 2f$.