**2. WASTE PROJECT (8 points)** In 2114, Europarliament decided that all radioactive wastes need to be sent to the Sun, so as to avoid contamination of Earth and orbital space. In what follows, you can use the following numerical data: duration of one year $T = 365.25\,\text{days}$, orbital speed on Earth $v_0 = 29.8\,\text{km/s}$, angular diameter of Sun as seen from the Earth $\alpha = 0.5^\circ$, radius of the Earth $R = 6400\,\text{km}$, free fall acceleration at the Earth's surface $g = 9.81\,\text{m/s}^2$.

According to the project, the waste is sent to the Sun using ballistic spaceships: the engine operates only during a short period of time during which the displacement of the spaceship remains much shorter than the radius of Earth. In the Earth's frame of reference, the spaceship obtains a velocity opposite to the orbital velocity of Earth in the Sun's frame of reference. Further, the ship moves along a ballistic trajectory until it hits the Sun. The trajectory is such as to minimize the consumption of fuel.

**i)** *(1 point)* Sketch the trajectory of the spaceship.

As a first approximation, calculations can be done when neglecting the angular size of the Sun (i.e. by putting $\alpha \approx 0^\circ$); you can use this approach for the next two questions.

**ii)** *(1.5 points)* How long will it take for the spaceship to travel from the Earth to the Sun?

**iii)** *(1.5 points)* What is the speed of the spaceship in the Earth's frame of reference when the distance from the Earth is much larger than the Earth's radius, but still much shorter than the distance to the Sun?

**iv)** *(2.5 points)* Answer the previous question without making the approximation $\alpha \approx 0^\circ$.

**v)** *(1.5 points)* What is the speed of the spaceship in the Earth's frame of reference when the distance from the Earth is much smaller than the Earth's radius?