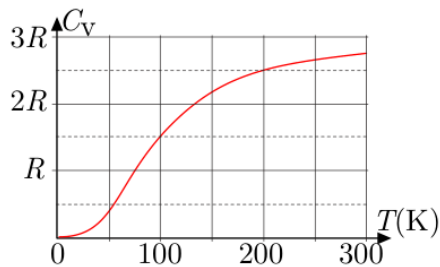
9. THERMAL ACCELERATION (9 points) Consider a cube of side length $a = 1\,\mathrm{cm}$, made of aluminium (density $\rho = 2.7\,\mathrm{g/cm^3}$, molar mass $M_A = 23\,\mathrm{g/mol}$). The heat capacitance of one mole of aluminium is given as a function of temperature in the graph below. The speed of light $c = 3 \times 10^8\,\mathrm{m/s}$, universal gas constant $R = 8.31\,\mathrm{J/(kg \cdot K)}$. The initial temperature of the cube is $T_0 = 300\,\mathrm{K}$.



i) *(1 point)* What is the total heat energy of such a cube at the initial temperature T_0?

ii) *(3 points)* Now, the cube has 5 faces painted in white (reflects all relevant wavelengths) and one face painted in black (absorbs all these waves). The cube is surrounded by vacuum at a very low temperature (near absolute zero); there is no gravity field. Initially, the cube is at rest; as it cools down due to heat radiation, it starts slowly moving. Estimate its terminal speed v_1.

iii) *(2 points)* At very low temperatures, the heat capacitance of aluminium is proportional to T^3, where T is its temperature. Which functional dependance $f(t)$ describes the temperature as a function of time [$T = A \cdot f(Bt)$, where A and B are constants] for such very low temperatures under the assumptions of the previous question?

iv) *(3 points)* Now, the cube has 5 faces covered with a thermal insulation layer (you may neglect heat transfer through these faces). One face is left uncovered. The cube is surrounded by hydrogen atmosphere at a very low temperature (molar mass of hydrogen molecules $M_H = 2\,\mathrm{g/mol}$). The cube starts cooling down due to heat transfer to the surrounding gas; you may neglect the heat radiation. Initially, the cube is at rest; as it cools down, it starts slowly moving. Estimate the order of magnitude of its terminal speed v_2. Assume that the surrounding gas is sparse, so that the mean free path of the molecules is much larger than a. Assume that $v_2 \ll c_s$ where c_s is the speed sound in the atmosphere surrounding the cube.