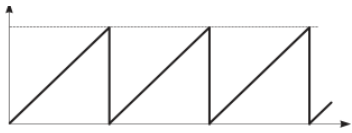
1. DC-DC converter

1) (1 pt) From the Kirchhoff's voltage law for the loop consisting of L and $\mathcal{E}$, $\mathcal{E} = L\frac{dI}{dt}$, hence $I = \mathcal{E}t/L$. From $I_0 = \mathcal{E}\tau_L/L$ we obtain

$$\tau_L = LI_0/\mathcal{E}.$$

2) (1 pts) Once the current I_0 is reached, the key is opened; the current trough L cannot change instantaneously and therefore is forced to flow through the resistor R. Since the characteristic time of this current loop (consisting of L and R is very short ($L/R \ll \tau_K$), the current decays very fast and becomes essentially equal to zero while the key is still open. Now there is no current through the inductor, so that the key will close again and the process will start repeating from the beginning. As a result we'll have a periodic graph as shown in figure.



3) (1 pt) The voltage through the resistor is maximal when the current is maximal, which happens immediately after the switch is opened; the maximal current is I_0 so that $V_{\max} = RI_0$.

4) (2 pts) Due to $V_{\max} \gg V_0$, we can neglect the effect of the diode; so we have the Kirchhoff's voltage law $L\frac{dI}{dt} = RI = R\frac{dq}{dt}$ (here we expressed the current via the charge q which flows through the resistor). Integration over a single cycle (when the inductor current drops from I_0 down to 0) yields $LI_0 = R\Delta q$, hence the charge flown through the resistor (and through the diode) $\Delta q = I_0 L/R$. During that cycle, the diode had a constant voltage V_0, so the electric field performed work $A = V_0\Delta q$ which was released as heat in the diode. So, the average power dissipation

$$P = \frac{A}{\tau_L} = \frac{V_0 I_0 L}{R\tau_L} = \frac{V_0\mathcal{E}}{R}.$$

5) (2 pts) Now, since the characteristic time of the RC-loop is very large, the capacitor maintains its charge (and voltage) during that period of time when the diode is closed. Once the key K_1 opens, the diode will open, and the capacitor is connected to the inductor so that a LC-loop is formed. That loop admits oscillations of period $T_{LC} = 2\pi\sqrt{LC}$ and as the current to the resistor can be neglected (since $RC \gg T_{LC}$), we can neglect the presence of the resistor. During the time interval when the diode is open, we can also neglect the presence of the diode since $V_0 \ll V_{\max}$. Hence, the current i in the LC-loop changes in time sinusoidally, starting from $i = I_0$ and ending when $i = 0$ (then the diode will close disconnecting the LC-loop). During that process, the magnetic energy of the inductor $\frac{1}{2}LI_0^2$ is converted into the electric field energy of the capacitor, which is later released as heat on the resistor. When the stationary regime is achieved, the energy lost by the capacitor during one period (of duration τ_L) as the heat dissipation on the resistor $Q = V_{av}^2\tau_L/R$ must be equal to the energy received from the inductor; so,

$$\frac{V_{av}^2\tau_L}{R} = \frac{1}{2}LI_0^2 \Rightarrow V_{av} = I_0\sqrt{\frac{LR}{2\tau_L}} = \sqrt{\frac{\mathcal{E}I_0R}{2}}.$$

6) (1 pt) The charge which flows away from the capacitor when the diode is closed can be found as $q_C = \tau_L V_{av}/R$ (owing to $RC \gg \tau_{LC}$, the relative change of the capacitor's voltage is small). Hence, the voltage drop is found as $\Delta V = q_C/C = \tau_L V_{av}/(RC)$. The amplitude is half of the difference between the minimal and maximal values, so that the amplitude

$$U_0 = \frac{\Delta V}{2} = \frac{\tau_L V_{av}}{2RC} = \frac{I_0 L}{2C}\sqrt{\frac{I_0}{2\mathcal{E}}}.$$