3. Magnets

There three forces acting on the hanging magnet: the downwards directed gravity force $m\vec{g}$, the tension force $\vec{T}$, which is directed along the thread, and the horizontal magnetic force $\vec{F}_{\mathrm{m}}$. Since the thread's angle is very small, the modulus of $\vec{T}$ is almost equal to mg, so that its horizontal projection is expressed as $-(x/l)mg$, where l is the length of the thread and $x = 1\,\mathrm{cm}$ (the displacement from the initial position). The net horizontal force $F = F_{\mathrm{m}} - (x/l)mg$. At the equilibrium point $F = 0$. The equilibrium point is stable if a small (virtual) displacement Δx gives rise to a returning force

$$\Delta F = \Delta F_{\mathrm{m}} - \frac{\Delta x}{l} mg$$

which needs to push towards the equilibrium point. Let $F_{\mathrm{m}} = kd^{-n}$, where k is an unknown proportionality coefficient. Then

$$\Delta F_{\mathrm{m}} = F'_{\mathrm{m}}(d)\Delta d = \frac{kn}{d^{n+1}}\Delta x,$$

because $\Delta d = -\Delta x$. Therefore

$$\Delta F = \left(\frac{kn}{d^{n+1}} - \frac{mg}{l}\right)\Delta x.$$

At the limit case of the loss of stability (which is described by this problem) $\Delta F = 0$. Thus we have two equations with two unknowns (n and k):

$$\frac{k}{d^n} - \frac{xmg}{l} = 0, \qquad \frac{kn}{d^{n+1}} - \frac{mg}{l} = 0;$$

this can be rewritten as

$$\frac{k}{d^n} = \frac{xmg}{l}, \qquad \frac{kn}{d^{n+1}} = \frac{mg}{l}.$$

If we divide the corresponding sides of the two equations we obtain $d/n = x$, hence

$$n = d/x = 4.$$