2. HOLOGRAPHIC LENS (7 points) *(by J. Kalda)* Monochromatic light can be focused into a point using an holographic lens (better known as a *Fresnel zone plate*). This is a thin film which has a system of opaque and transparent concentric rings: the opaque rings obstruct those light waves which would arrive at the focus in the opposite phase (as compared with the light from the transparent rings). In what follows, let us assume that the diameter and focal length of such a lens $d = f = 10\,\mathrm{cm}$; the wavelength of the monochromatic incident light radiation is $\lambda = 5 \times 10^{-7}\,\mathrm{m}$.

i) *(1.5 points)* Starting enumeration from the (opaque) centre of the lens, what is the radius of the m-th transparent ring (more precisely, the circle at the middle of the ring)?

ii) *(1.5 points)* Let us consider also a glass lens of the same focal length f and diameter d, such that a parallel incident beam is collected perfectly into the focus (has aspheric surfaces). What is the minimal possible thickness of such a lens if the coefficient of refraction of the glass is $n = 1.5$?

iii) *(2 points)* If a short light pulse falls onto a lens, the behaviour of the holographic lens is very different from that of the glass lens. Indeed, if we ignore the dispersion of the glass (the dependence of n on the wavelength), the pulse duration at the focus of the glass lens is the same as that of the incident beam. Meanwhile, in the case of a short pulse ($\tau = 3 \times 10^{-14}\,\mathrm{s}$), the pulse duration at the focus of the holographic lens becomes significantly dilated. Sketch qualitatively the light intensity I at the focus of the holographic lens as a function of time. A scale is required for time t, but not for I. The speed of light $c = 3 \times 10^8\,\mathrm{m/s}$.

iv) *(1 point)* Even if the light source is an ideal laser providing a perfectly monochromatic radiation, the short pulse is no longer strictly monochromatic. Estimate the width of the range of wavelengths in the pulse of duration τ.

v) *(1 point)* Estimate the duration of this light pulse in the focus of the glass lens. Assume that in the glass, the light wave group velocity v_g depends on the wavelength at the rate $\frac{\mathrm{d}v_g}{\mathrm{d}\lambda} = 0.02 \times \frac{v_g}{\lambda}$ and that for the wavelength λ, group speed is equal to the phase speed.