4. STAR WARS (8 points) *(by J. Kalda)* Here we'll study, in which region of space targets can be hit with an interplanetary ballistic missile. The launching point P is motionless in the inertial frame of reference of a gravitationally attracting star centred at S. The launching speed of the missile is fixed throughout this problem and equal to v_0, the same applies to the distance to launching point $|SP| = R$. The mass of the star is M, and the gravity constant is G. *Hint:* the total energy of a missile of mass m moving on an elliptical orbit is $E = -GMm/(2a)$, where a is the longer semiaxis of its orbit.

i) *(2 points)* The missile is launched as described above in an arbitrary direction. What is the period of the missile on its elliptic orbit?

ii) *(2 points)* According to the Kepler's I law, the missile's orbit is an ellipse, with one of the foci being at S. The position of the other focus F depends on the launching angle. While the launching angle is changed, the focus F moves along a certain curve; what curve is it? Also, determine the geometric dimensions of that curve.

iii) *(1 point)* Let us consider such a point Q on the missile's trajectory, the distance of which from the star equals to $|SQ| = r$. Determine the distance $|QF|$.

iv) *(1 point)* The position of point Q (as defined above) depends on the launching angle of the missile, and so does the distance $|PQ|$. Among all the possible positions of the point Q, what is the largest possible distance $|PQ|$?

v) *(2 points)* For questions iii and iv, we assumed that the distance r had a fixed value. Now, if we consider r as a free parameter, the position of the point Q is a function of both r and of the launching angle. What is the boundary of that region of space to which the point Q can belong?