1. ANNIHILATION

i) Electron's total energy $E = \gamma m_e c^2 \implies \gamma = \frac{E}{m_e c^2}$, where $E = T + m_e c^2$ (here, $T = 1\,\mathrm{MeV}$ is the given kinetic energy). Now, $\gamma = 1/\sqrt{1 - v_e^2/c^2} \implies v_e = c\sqrt{1 - 1/\gamma^2} = c\sqrt{1 - m_e^2 c^4/E^2} = \frac{c\sqrt{T^2 + 2Tm_e c^2}}{T + m_e c^2}$. Numerically, $v_e \approx 0.941c$.

ii) The photons fly away symmetrically with respect to the electron's trajectory: in the zero-total-momentum frame, momentum conservation implies that the photons have equal momenta and, thus, equal energies, and fly in exactly opposite directions; their energies can be equal also in the positron's frame only if they are flying totally symmetrically. Each photon gets a half of the total energy in the system: $E_\gamma = \frac{1}{2}(T + 2m_e c^2) \approx 1.01\,\mathrm{MeV}$.

iii) $E_\gamma = p_\gamma c \implies p_\gamma = E_\gamma/c = 1.01\,\mathrm{MeV}/c$.

iv) From $E = p_e^2 c^2 + m_e^2 c^4$, the electron's momentum is $p_e = \sqrt{\frac{E^2}{c^2} - m_e^2 c^2} = \frac{1}{c}\sqrt{T^2 + 2Tm_e c}$ (Equivalent result can be derived from $p_e = \gamma m_e v_e$.) Momentum conservation in the z-direction then implies that $p_e = 2p_\gamma \cos\alpha \implies \alpha = \arccos\frac{p_e}{2p_\gamma} = \arccos\frac{1}{\sqrt{1 + 2m_e c^2/T}} = \arctan\sqrt{\frac{2m_e c^2}{T}} \approx 45.3°$.

v) In the center-of-mass frame, the total momentum of any system is zero. This means that if the outcome of a collision is only a single particle, then the particle's momentum must be zero in the center-of-mass frame. However, a photon's momentum can never be zero, because otherwise it would have zero energy and an infinite wavelength.