4. STAR WARS

i) The period of a Keplerian orbit having major semi-axis a can be expressed using relation $T^2 = \frac{4\pi^2 a^3}{GM}$. However, if we didn't know the formula, we should recall Kepler's 3rd law $T^2 \sim a^3$ and simply express the period for a circular orbit with $a = r$. Therefore, $T^2 = \frac{4\pi^2 r^2}{v^2}$, were we need to insert v from $\frac{v^2}{r} = \frac{GM}{r^2}$.

For finding the period, we can get the major semi-axis from the expression for orbit's total energy $-\frac{GMm}{2a} = \frac{mv^2}{2} - \frac{GMm}{R}$, equivalently $a = \frac{GM}{2\frac{GM}{R} - v^2} = \frac{1}{\frac{2}{R} - \frac{v^2}{GM}}$. Finally $T = \frac{2\pi a^{3/2}}{\sqrt{GM}}$.

ii) According to the properties of an ellipse, the distances l_1 and l_2 drawn from any point on the ellipse to its two foci add up to constant value $l_1 + l_2 = 2a$. Since $l_1 = R$, the other focus must be at distance $l_2 = 2a - R$, so the other focus F lies on a circle with radius $|PF| = l_2 = 2a - R$ around P.

iii) This is the previous part in reverse. Since we have distance to one focus given $|QS| = r$, we can express the distance to the other focus $|QF| = 2a - r$.

iv) The points P, Q, and F form a triangle, for which we have expressed two of its side lengths: $|PF|$ and $|QF|$. The third side QP must satisfy $|QP| \leq |PF| + |QF|$. The $|QP|_{\max}$ is obtained for a degenerated triangle (all the three points lying along a common line). Therefore, $|QP|_{\max} = 4a - R - r$.

v) It turns out that for all the maximal distance points Q the sum of $|SQ| + |PQ|$ is constant and equal to $4a - R$. That means it also defines an ellipse with focal points S and P and major semi-axis $2a - R/2$. This ellipse is the area that we can hit.