8. ELECTRIC FIELD

i) By the Biot-Savart law, every piece of the ring with length and direction $d\vec{l}$ creates a magnetic field $d\vec{B} = \frac{\mu_0 I\, d\vec{l}\times\hat{a}}{4\pi a^2}$, where $\vec{a}$ is the displacement vector from the piece to the point where we are calculating $\vec{B}$, and $\hat{a} = \vec{a}/a$. By symmetry, the magnetic field on the z-axis adds up to being exactly along the z-axis, because the perpendicular components cancel out. dB_z is proportional to dl and $d\vec{l} \perp \hat{a}$. The vector $d\vec{l} \times \hat{a}$ is at an angle $\alpha = \arctan\frac{r}{z}$ from the horizontal. Therefore we can add up the contributions and write $B(z) = \frac{\mu_0 I \times 2\pi R \sin\alpha}{4\pi(z^2+R^2)} = \frac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}}$. The field is along the z-axis.

ii) The electric field measured by a charged observer must be in the same direction as the Lorentz force acting on the observer in the laboratory frame of reference. Therefore the electric fieldlines are circles around the z-axis. Now we may apply Faraday's induction law to such a circle, as it approaches the ring: the total electromotive force $E(z,r)\times 2\pi r = \dot{\Phi} = -\frac{d\Phi}{dz}v \approx -\frac{d}{dz}B(z)\times\pi r^2 \times v \Longrightarrow E(z,r) = -\frac{rv}{2}B'(z) = \frac{3\mu_0 I r R^2 v z}{4(z^2+R^2)^{5/2}}$.